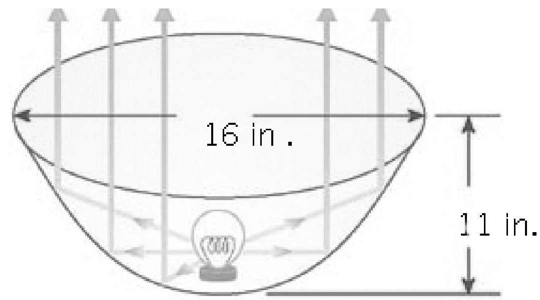
Math 5 – Trigonometry – Chapter 6 Test Problems –

1. A light bulb is to be placed at the focus of a parabolic dish as shown in the figure at right. How high above the bottom should the light be placed?



2. Find an equation for the ellipse with foci $(\pm 10, 0)$ and vertices $(\pm 11, 0)$.

3. Find the vertices, foci, and asymptotes of the hyperbola $144y^2 - 36x^2 = 3600$ and sketch a graph illustrating these features.

4. Find an equation for the hyperbola with asymptotes $y = \pm\frac{2}{3}x$ and vertices at
   a. $(0, \pm 3)$
   b. $(\pm 3, 0)$

5. Complete the square to determine whether the equation represents an ellipse, a parabola, or a hyperbola. If the graph is an ellipse, find the center, foci, eccentricity and endpoints of the major and minor axes. If it is a parabola, find the vertex, focus and directrix. If it is a hyperbola, find the center, foci, vertices and asymptotes.
   a. $16x^2 - 96x + 9y^2 = 0$
   b. $36y^2 - x^2 - 8x - 52 = 0$
   c. $x^2 - 8x - 32y - 240 = 0$

6. If the coordinate axes are rotated through an angle of 60°. Find the new coordinates of the point (3,5).

7. Use the discriminant to determine whether $2y^2 + 5xy + 2x^2 = 4x$ describes a parabola, ellipse or hyperbola.

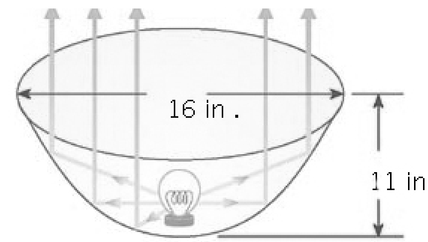
8. Find the angle of rotation of axes to eliminate the xy – term in the following equations. Write the angle in radians and approximate with 4 significant digits.
   a. $11x^2 - 24xy + 4y^2 + 15 = 0$
   b. $2\sqrt{3}x^2 - 6xy + 12y + 4\sqrt{3}x = 0$

9. Find parametric equations to describe the hyperbola $4(x-1)^2 - 25(y-4)^2 = 100$

10. Write the equation for the conic section described by $\begin{matrix} y = 1 + 8\sin(2t) \\ x = 9 + 72\cos(2t) \end{matrix}$ in rectangular form.

Math 5 – Trigonometry – Chapter 6 Test Solutions.

1. A light bulb is to be placed at the focus of a parabolic dish as shown in the figure at right. How high above the bottom should the light be placed?
   SOLN: If the parabola is opening upwards from a vertex at (0,0), then it has the form $4py = x^2$, whence $44p = 64$ and the distance from the focus to the vertex is $p = 16/11$.

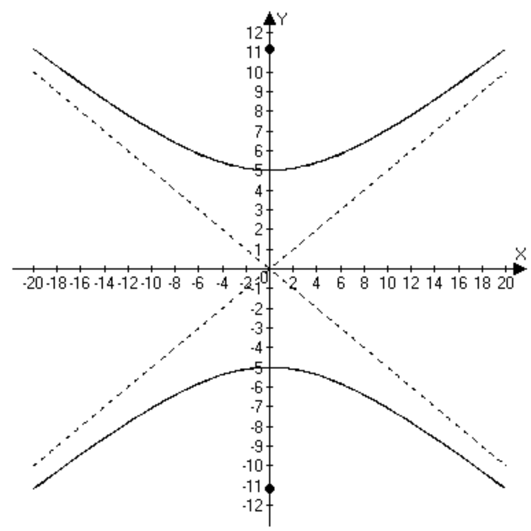


2. Find an equation for the ellipse with foci $(\pm 10, 0)$ and vertices $(\pm 11, 0)$.

   SOLN: $b^2 = a^2 - c^2 = 11^2 - 10^2 = 21$ so the equation is $\frac{x^2}{100} + \frac{y^2}{21} = 1$

3. Find the vertices, foci, and asymptotes of the hyperbola $144y^2 - 36x^2 = 3600$ and sketch a graph illustrating these features.

   SOLN: $144y^2 - 36x^2 = 3600 \Leftrightarrow \frac{y^2}{25} - \frac{x^2}{100} = 1$ has vertices at foci at $(0, \pm 5)$ $\left(0, \pm 5\sqrt{5}\right)$ The asymptotes are $y = \pm \frac{1}{2}x$



4. Find an equation for the hyperbola with asymptotes $y = \pm \frac{2}{3}x$ and vertices at

   a. $(0, \pm 3)$ So we know the ratio of $b/a = 2/3$ and that $b = 3$. Thus $a = 9/2$ and the equation is $\frac{y^2}{9} - \frac{4x^2}{81} = 1$

   b. $(\pm 3, 0)$ Here $a = 3$ so $b = 2$ and the equation is simply $\frac{x^2}{9} - \frac{x^2}{4} = 1$

5. Complete the square to determine whether the equation represents an ellipse, a parabola, or a hyperbola. If the graph is an ellipse, find the center, foci, eccentricity and endpoints of the major and minor axes. If it is a parabola, find the vertex, focus and directrix. If it is a hyperbola, find the center, foci, vertices and asymptotes.

   a. $16x^2 - 96x + 9y^2 = 0 \Leftrightarrow 16\left(x^2 - 6x + 9\right) + 9y^2 = 144 \Leftrightarrow \frac{(x-3)^2}{9} + \frac{y^2}{16} = 1$ is an ellipse with center (3,0), endpoints of minor axes at (0,0) and (6,0) and major axes at (3, –4) and (3, 4) with foci at $\left(3, \pm\sqrt{7}\right)$ and eccentricity $\sqrt{7}/4$

b. $36y^2 - x^2 - 8x - 52 = 0 \Leftrightarrow 36y^2 - (x-4)^2 = 36 \Leftrightarrow y^2 - \dfrac{(x-4)^2}{36} = 1$ is an ellipse with center (-4,0), vetices (-4,–1) and (-4,1), foci at $\left(-4, \pm\sqrt{37}\right)$ and asymptotes $y = \pm\dfrac{1}{6}(x-4)$.

c. $x^2 - 8x - 32y - 240 = 0 \Leftrightarrow 32(y+8) = (x-4)^2$ is a parabola with vertex (4, –8), focus at (4,0), directrix along $y = -16$.

6. If the coordinate axes are rotated through an angle of 60°. Find the new coordinates of the point (3,5).

SOLN: $u = (\cos 60^\circ)x + (\sin 60^\circ)y = \dfrac{3}{2} + \dfrac{5\sqrt{3}}{2}$, $v = -(\sin 60^\circ)x + (\cos 60^\circ)y = \dfrac{-3\sqrt{3}}{2} + \dfrac{5}{2}$

7. Use the discriminant to determine whether $2y^2 + 5xy + 2x^2 = 4x$ describes a parabola, ellipse or hyperbola.
SOLN: $B^2 - 4AC = 25 - 16 > 0$ so it's a hyperbola.

8. Find the angle of rotation of axes to eliminate the xy – term in the following equations. Write the angle in radians and approximate with 4 significant digits.

a. $11x^2 - 24xy + 4y^2 + 15 = 0$

SOLN: $\phi = \dfrac{1}{2}\arctan\left(\dfrac{-24}{11-4}\right) = -\dfrac{1}{2}\arctan\left(\dfrac{24}{7}\right) \approx -0.6435$

b. $2\sqrt{3}x^2 - 6xy + 12y + 4\sqrt{3}x = 0$

SOLN: $\phi = \dfrac{1}{2}\arctan\left(\dfrac{-6}{2\sqrt{3} - 12}\right) \approx 0.3063$

9. Find parametric equations to describe the hyperbola $4(x-1)^2 - 25(y-4)^2 = 100$

SOLN: $x = 1 + 5\sec t$ and $y = 4 + 4\tan t$

10. Write the equation for the conic section described by $\begin{array}{l} y = 1 + 8\sin(2t) \\ x = 9 + 72\cos(2t) \end{array}$ in rectangular form.

$$\frac{(x-9)^2}{5184} + \frac{(y-1)^2}{64} = 1$$